Algorithms for the Analysis of the Nystagmic Eye Movements Induced by Sinusoidal Head Rotations

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Abstract—**A digital computer program has been developed for analysis of the nystagmus which results from sinusoidal head rotation. For any sinusoidal vestibular stimulation between 0.05 Hz and 1 Hz, the program, accepts sampled records of eye position and head velocity and yields cumulative eye position as well the phase and the gain of the vestibuloocular reflex. Described in this paper are new algorithms by which fast phases are detected and by which slow phase cumulative eye position is reconstructed using and iterative procedure of best least squares fit. These algorithms are useful in neurophysiological studies on normal animals, but not in clinical analysis of nystagmus.**

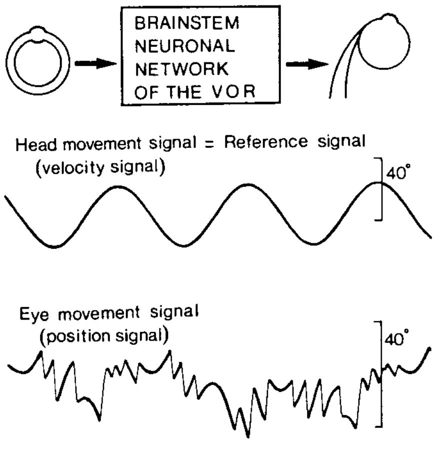

Fig. 1. Vestibular nystagmus during sinusoidal rotation of the head (top). Sketch of the vestibuloocular reflex (VOR) (bottom). Velocity of the head and resulting position of the eye as a function of time.

I. Introduction

In the dark, any rotation of the head in one direction generates automatically an eye movement in the opposite direction. The input signal, the angular acceleration of the head, is sensed by the semicircular canals. From the canals the signal is further processed by a brainstem neuronal network. The output of the system is the eye movement. The reflex, as a whole, is known as the vestibuloocular reflex (VOR) (Fig. 1) [1].

In fact, during head rotations, the compensatory movements of the eye are interrupted from time to time by fast flicks in the direction of rotation. As a result, the eye movement as a function of time has a characteristic sawtooth appearance consisting of *slow phases* in the compensatory direction and *quick phases* in the anticompensatory direction. This whole eye movement is known as the *nystagmus* (Fig 1).

Of course, investigators interested primarily in the ocular movements induced by a vestibular stimulus want to analyze eye movements which would be generated if quick phases did not occur. Thus, the aim of many (included ours) nystagmus algorithms is to remove automatically the effect of the quick phases.

Previous nystagmus algorithms to yield slow phase cumulative eye position [2]–[5] were based on the following two main considerations. 1) The selection criterion to distinguish slow phases from fast phases was the velocity of the eye movement. 2) Reconstruction of the slow phase cumulative eye position was made by polynomial first- to third-order extrapolation of the slow phase movement across the fast phases.

These algorithms work properly provided that the velocity of the stimulus is not too high. If the velocity is too high, the "slow" phases are not slow enough and correct selection becomes difficult. Furthermore, simple polynominal extrapolation induces errors when the velocity of the "slow" eye movement changes rapidly as with large sinusoidal vestibular stimulations.

The objective of this paper is to propose a new kind of algorithm to reconstruct the slow phase response when animals are subjected to sinusoidal rotations of 20° amplitude ranging from 0.5 Hz to 1 Hz (maximal velocity ranging from 6.2 to 125.6°/s.

II. Rationale for Present Approach

Knowledge of the frequency response of a linear system enables one to predict the output resulting from any conceivable imput [1], [6]. The vestibuloocular reflex is a biological system which is linear over a relatively large range [7], [8]. Therefore, it is useful to describe this system in terms of its frequency response, using sinusoidal rotations in the dark at different frequencies.

Our algorithm assumes the slow phase eye response resulting from a sinusoidal rotation of the head to be sinusoidal. Such a representation is accurate and meaningful. Indeed, when quick phases are lacking, as with small sin-

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usoidal rotation and in decerebrate animals [9], the resultant eye movements are sinusoidal. Larger head movements in normal, awake animals produce, by the vestibuloocular reflex, eye movements which are interrupted by quick phases. But evidence exists which favors the idea that quick phases do not disrupt the basically sinusoidal pattern of the response and only produce brief displacements of the whole record. It is possible to remove manually the quick phases from the raw records [10]. This is done by continuing the end of each slow phase during the duration of the next quick phase, the curvature of the slow phase being taken into account in each case. The beginning of the next slow phase is then put on this extrapolated point. The slow phase response isolated by this procedure is quasi-sinuosidal [10]. We justified here our basic assumption. Its advantages and limitations will be discussed later (see Section IV).

Either an eye movement completely unrelated to the vestibular stimulation or an artifact can occur during a slow phase and distorts it. Such distorted phases will be referred to as "bad" slow phases and one of the objectives of our algorithm is to exclude them from the calculated slow phase response. Throughout the paper "bad" slow phases are set against the "good" ones.

III. Recording, Stimulating, and Computer Devices

The head of the cat was put in the center of the turntable and placed so that the horizontal semicircular canals were horizontal. The movement of the turntable was monitored by an ac tachogenerator. The amplitude of the sinusoidal movement of the turntable was 20°. Frequencies ranged from 0.05 to 1 Hz. Horizontal eye movements were recorded using the scleral search coil technique [11]. The signals were analyzed off-line. Algorithms were written in Fortran 77[1] and run on a Digital Equipment Corporation PDP 11/23. The rate of sampling varied with the tested frequency of the sinusoidal stimulation. It was 66/s in the range 0.05 Hz–0.25 and 200/s in the range 0.5 Hz–1 Hz.

IV. Description of Algorithms

A. Overview

Basically, the algorithm fits the slow phases onto a sinusoid waveform whose frequency is that of the head movement. The steps of processing used in evaluation of the VOR response to a sinusoidal rotation of the head are outlined in Fig. 1.

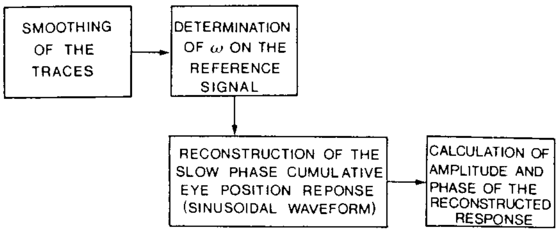

Fig. 2. Processing steps for evaluation of the vestibuloocular response induced by a sinusoidal rotation of the head. The reconstruction of the slow phase cumulative eye position response, step three, is detailed in Figs. 3 and 4.

First, the head movement (velocity signal) and the eye movement (position signal) signals are smoothed (Fig. 2). *Second*, the frequency of the sinusoidal stimulation is determined from the head movement signal with a 0.25 percent precision, the precision of the driving system (two percent) being estimated not to be enough. *Third*, the pure VOR response is reconstructed from the eye movement signal. This response is assumed to be sinusoidal. The method selects the slow phases and fits them onto a sinusoidal waveform whose frequency is that of the head movement. A first selection of slow phases is done using an approximative criterion. A best least squares fit is done on these selected phases. The error of fit of each of these phases is then calculated. The phase with the largest error is rejected if this error is too high. The fit is redone with the remaining phases. This iterative procedure is carried out until it rejects no more phases. In the next step, the algorithm scans all the phases of the data and rejects the fast phases on the basis of a too large error of fit. The definitive fit is carried out with all the slow phases, using the same iterative procedure as that used in the approximative fit. As a result, all the "bad" slow phases are rejected out of the fit. *Fourth*, comparison of the reconstructed eye movement signal with the head position signal provides the gain and the phase of the VOR response (Fig. 2).

Figs. 3 and 4 illustrate in detail the sequence of processing events occurring in the reconstruction of the pure sinusoidal VOR response. Fig. 3 is concerned with a low-frequency stimulus (0.05 Hz), Fig. 4 with a high-frequency stimulus (1 Hz).

B. Smoothing

Smoothing is executed in three steps. The ordinate value of each point is readjusted by a best least squares fit of that point and its neighbors on a second-order polynomial. This procedure is carried out three times. Four neighbor points are included in the first two runs, two in the third run. Both the eye and head movement signals are smoothed.

C. Fit of the Reference Signal

The reference signal is a sinusoidal movement of the turntable. Its angular frequency ω_s is established to a precision of two percent. The best least squares fit of the reference signal is calculated by minimizing the function χ^2:

$$\chi^2 = \sum_{k=1}^{K} \left[y_k - (\alpha_r \sin \omega t_k + \beta_r \cos \omega t_k + C)\right]^2$$

where k is any point of the signal, K is the total number of points, α_r and β_r are the parameters of the sinusoid fitted on the reference signal, ω is the angular frequency,

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[1] Information about the Program written in Fortran 77 can be obtained from the authors.

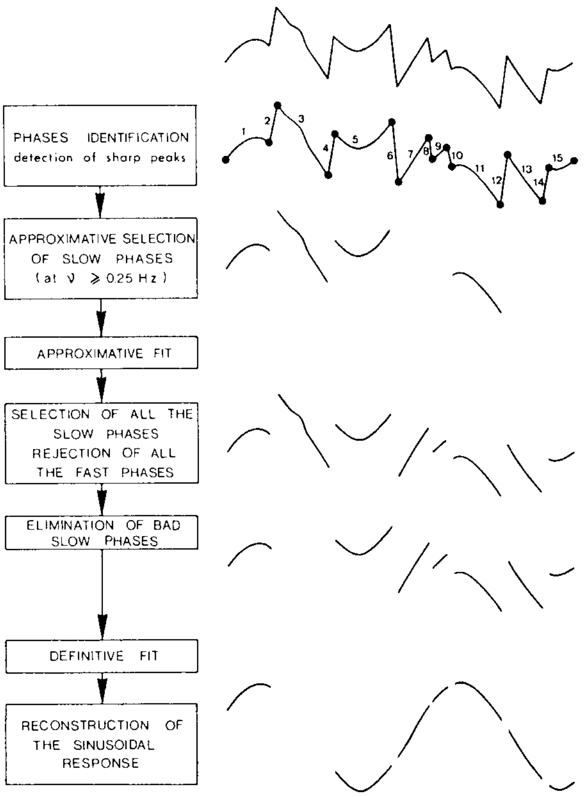

Fig. 3. Illustration of the various steps used in reconstruction of the slow phase cumulative eye position when the frequency of the sinusoidal stimulation is between 0.05 Hz and 0.25 Hz. The function of each step is described in the left column and illustrated in the right column. The first step detects the boundaries between the successive phases which receive a running number (1 to 15 in this simulated example). Approximate selection is then done according to a duration criterion: a phase is assumed to be slow if its duration is long. In this example, the algorithm selects phases 1, 3, 5, and 11 as slow phases. Notice that phase number 3 is a "bad" slow phase, distorted by an artifact. An approximative fit is then done. After, each phase (1 to 15) is considered and its error with respect to the preceding fit is calculated. Both quick phases (phase number 2, 4, 6, 8, 10, 12, and 14) and bad slow phases (phase number 3) are rejected. After the rejection of a phase, a fit is redone to which the remaining phases will be compared.

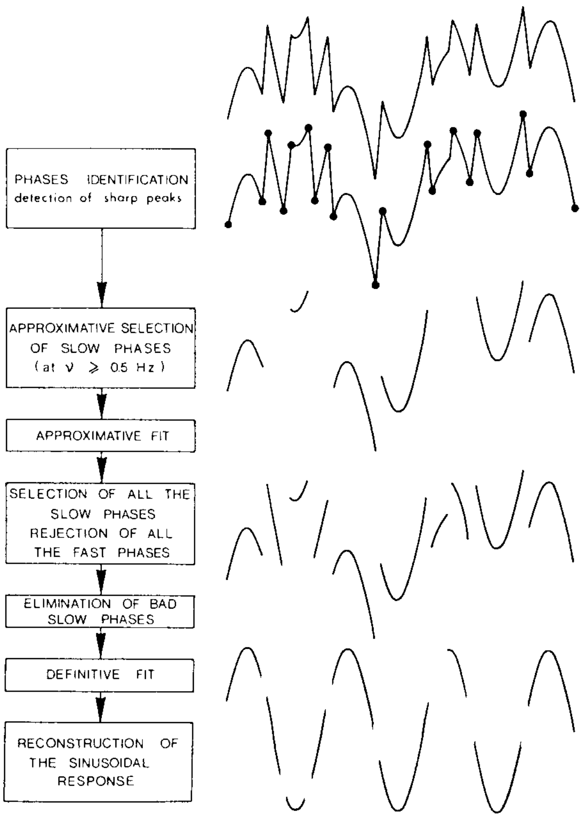

Fig. 4. Illustration of the successive steps of the reconstruction algorithm when the frequency of the sinusoidal rotation is between 0.5 Hz and 1 Hz. The organization of the figure is the same as that of Fig. 3. The steps of the algorithm are the same in both figures. The only difference is the approximative selection criterion: a phase is here assumed to be slow if it contains either a horizontal summit or a horizontal valley.

y_k is the amplitude of the signal at point k, t_k is the time of occurrence of point k, and C is a fitted parameter.

The function χ^2 is calculated for values of ω ranging from 0.98 ω_s to 1.02 ω_s. The value of ω for which χ^2 is minimum is assumed to be the accurate value of ω. From here reference to an ω value will be to that ω. It is calculated with a precision of 0.25 percent.

D. Phases Identification

Smoothed eye movement data are read and scanned for the occurrences of sharp peaks. A sharp peak corresponds to the boundary between two successive phases. First of all, the algorithm detects critical points as points where the derivative from the left and the derivative from the right are of different signs. When both derivatives from the left and the right are nearly zero (in fact, inferior to a threshold value), the critical point is either a horizontal summit or a horizontal valley, otherwise it is a sharp peak (see Figs. 3 and 4). The threshold value is chosen as the half of the mean value of the derivative calculated in each point of the trace. As a result, the threshold value is related to the quality of the signal.

E. Approximative Selection of the Slow Phases

How this step is processed depends on the frequency of the sinusoidal stimulation.

At frequencies between 0.05 Hz and 0.25 Hz, the phases are classified according to their durations. The longest, 33 percent (to a maximum of 30) are, in a first approximation, assumed to be slow phases. In fact, some fast phases and some "bad" slow phases are included in this first selection, but will be rejected later. Likewise, some good, but short slow phases that are excluded from the first choice will be reincluded in a further step (Fig. 3).

At frequencies between 0.5 Hz and 1 Hz, a phase is considered as a slow phase if it contains either an horizontal summit or an horizontal valley located between its boundary points (Fig. 4).

F. Approximate Fit

The data to be processed by a least squares fit are the selected slow phases, that is, pieces of sinusoid separated along the abscissa axis by varying time spaces and shifted along the ordinate axis by independent constants (Fig. 1).

The algorithm minimizes the function χ^2:

$$\chi^2 = \sum_{i=1}^{N} \sum_{j=1}^{M} [y_j - (\alpha \sin \omega t_j + \beta \cos \omega t_j + h_i)]^2$$

where i is the running number of any phase, N is the total number of phases processed, j is any point of any phase number i, M is the number of points in phase number i, y_j is the data, α and β are the parameters of the fitted sinusoid, h_i is the shift parameter of phase number i, and ω is the angular frequency of the sinusoid. ω is known from analysis of head rotation signal (see above).

The mean error of fit (MEF) is then calculated for each selected phase:

$$(\mathrm{MEF})^2 = \frac{1}{M} \sum_{j=1}^{M} [y_j - (\alpha \sin \omega t_j + \beta \cos \omega t_j + h_i)]^2$$

where α, β, and h_i are the coefficients calculated in the previous best fit and where j, M, y_j, ω, t_j have the same meaning as in the function χ^2 (see above). The phase with the largest $(\mathrm{MEF})^2$ is then rejected if

$$(\mathrm{MEF})^2 > 3\,(SD)^2$$

where SD is the typical error on the measurement of any point. The fit is redone with the remaining phases. This iterative procedure is carried out until it does not reject any more phase. At the end of this step, all the remaining phases are "good" slow phases. However, other good unselected slow phases are still outside the fit.

G. Quick Phase Rejection

In this step, all the phases of the signal, slow and quick, are taken into account. For each of them, the mean error of fit (MEF) is calculated with respect to the corresponding part of the previously fitted sinusoid, with the liberty that this part can be shifted anywhere along the ordinate axis. $(\mathrm{MEF})^2$ is defined as the minimum of the following expression:

$$\frac{1}{M} \sum_{j=1}^{M} [y_j - (\alpha \sin \omega t_j + \beta \cos \omega t_j + h_i)]^2$$

where α and β are the coefficients calculated at the end of the approximative fit, h_i is a parameter determined by the present minimization. A phase is rejected if

$$(\mathrm{MEF})^2 > 4.5\,(SD)^2$$

where SD is the typical error in the measurement of any point.

Usually, a quick phase will go in the direction opposite to that of the previous slow phase. As a result the preceding criterion is enough to reject all the quick phases (Figs. 3 and 4).

H. Definitive Fit

The iterative best least squares fit is done again, using all the slow phases, the good ones, and a few bad ones distorted by artifacts. The same criterion as that used in the approximative fit

$$(\mathrm{MEF})^2 > 3\,(SD)^2$$

reject the bad slow phases. Finally, the definitive fit is calculated, using only but all of the good slow phases (Figs. 3 and 4).

I. Visualization of the Reconstructed Response

The fitted curve is a sinusoid shifted in pieces. Cancellation of all the shift parameters allow us to visualize the reconstructed sinusoidal response.

V. RESULTS

Figs. 5 and 6 illustrate an example of reconstruction by our program of the slow phase response from the raw response induced by a "low"-frequency stimulus (0.05 Hz in Fig. 5) and by a "high"-frequency stimulus (1 Hz in Fig. 6).

The reconstructed sinusoidal data are analyzed further for establishing the diagram of the gain and phase of the VOR as a function of the frequency (Bode plot). Gain is defined as the ratio: peak-to-peak eye position/peak-to-peak rotating head position. Phase shift is designated as zero when eye and head movements are exactly opposite. Fig. 7 illustrates an example of a Bode plot of the vestibuloocular reflex computed by our program. Each point corresponds to the mean of six computings from raw sinusoidal nystagmus recorded on six successive days in the same cat.

VI. DISCUSSION

A. Comparison with Previous Algorithms

Previous nystagmus algorithms were based on the following two main considerations; 1) the selection criterion to distinguish slow phases from quick phases was the velocity of the eye movement and 2) reconstruction of the total slow phase position was made by polynomial extrapolation of the slow phase movement across the quick phases [2]–[5].

Our nystagmus algorithm is completely different. The VOR response to a sinusoidal rotation of the head is assumed to be sinusoidal. This response is reconstructed by an iterative procedure of best least squares fit. Both the selection of good slow phases and the reconstruction of the response are done progressively and in parallel.

B. Limitations of the Present Algorithm

Our algorithm assumes the VOR response to a sinusoidal movement of the head to be sinusoidal. It was explained in Section II that this is justified for normal animals and normal human beings. Of course, when either the vestibular system or the quick phases are impaired by disorders, assumptions that VOR response to a sinusoidal movement of the head is sinusoidal and that quick phases only produce brief displacements of the whole record are inaccurate. Therefore, our algorithm cannot be used for clinical analysis of nystagmus.

C. Applications of the Present Algorithm

Numerous investigators are interested by the study of the firing discharges of brainstem neurons during the

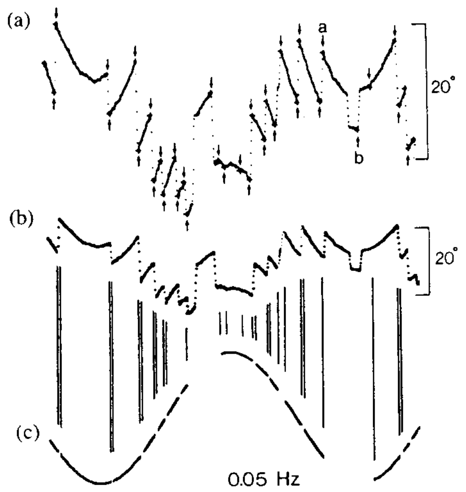

Fig. 5. Example of processing of the VOR response to a sinusoidal rotation the amplitude and the frequency of which are 20° and 0.05 Hz, respectively. (a) and (b), raw record at two different amplifications from which is reconstructed the slow phase cumulative eye position (c). Small arrows mark points detected by the algorithm as sharp peaks, as boundaries between phases. The piece of record between sharp peak *a* and sharp beak *b* is taken by the program only as a slow phase because there is no sharp peak between the slow phase and the next quick phase. But this wrong slow phase will be rejected later out of the fit as a "bad" phase, as a distorted phase.

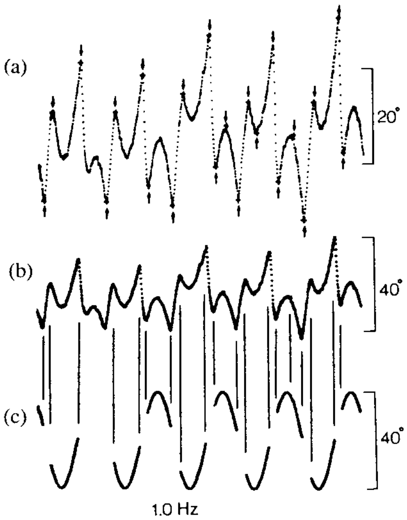

Fig. 6. Example of processing of the VOR response to a sinusoidal rotation the amplitude and the frequency of which are 20° and 1 Hz, respectively. (a) and (b), raw record at two different amplifications from which the slow phase cumulative eye position is reconstructed (c).

VOR. They want to correlate the firing rate of neurons with the time course of sinusoidal stimulation. Quick phases and related changes of firing rate of the neurons prevent them from doing that easily. Several cycles must be averaged. Each of them is divided into equal time bins, the number of spikes occurring in each bin being counted. The user then rejects manually, one by one, those bins that were contaminated by a transient discharge associated with a quick phase by pointing out the beginning and the end of each saccade with a joystick controlled cursor [12]. Our algorithm selecting the good slow phases would be able to do this procedure automatically.

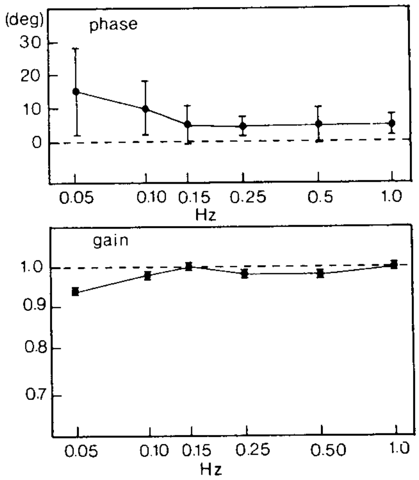

Fig. 7. Gain and phase of the vestibuloocular reflex as a function of frequency (Bode plot). Each point corresponds to the mean of six computings from raw sinusoidal nystagmus recorded on six successive days in the same cat. Vertical bars correspond to standard deviations.

Since the work of Gonshor and Melvill Jones [13], the interest of many investigators has been directed to the study of the adaptive modifications of the VOR [14]. This kind of study necessitates repetitive measurement of gain and phase. When the adaptive modification is a decrease of the gain, testing with large sinusoidal movements is necessary to get a better precision. Our algorithm meets all the requirements of this type of study. It is accurate as the response of the adapted animal to a sinusoidal rotation is well known to be sinusoidal [14].

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